Cosmology With Axionic-quintessence Coupled with Dark Matter

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We study the possibility of explaining the late time acceleration with an axion field which is coupled with the dark matter sector of the energy budget of the Universe. The axion field arises from the Ramond-Ramond sector of the Type-IIB string theory. We study the background evolution of the Universe as well as the growth of the matter perturbation in the linear regime. We subsequently use the observational data from Sn-Ia, BAO measurements, measurements of the Hubble parameter as well as the observational data for the growth of the matter perturbation to constrain our model. Our results show that coupled axion models are allowed to have larger deviation from cosmological constant by the present observational data.

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I. INTRODUCTION

Cosmological observations [1–4] have now established the fact that our universe is undergoing a late time accelerating phase on large scales. This has been confirmed by a number of precise cosmological observations. One generally believes that an unknown component having a large negative pressure is driving this acceleration and efforts are on to explain this acceleration by suitable modification of gravity at large cosmological distances. For a detailed review on various model building aspects for this late time acceleration, we refer the readers to some excellent reviews on this subject [5].

If the acceleration is indeed driven by some unknown candidate, then a cosmological constant Λ with an equation of state (e.o.s) $w = -1$ is the simplest option. But the model is plagued by two very important theoretical issues: the fine tuning problem and the cosmic coincidence problem. Moreover, the observational data itself can allow models which are different from a constant Λ. These are termed as quintessence models [6], and are primarily built with scalar fields with sufficiently flat potentials (Please see Copeland et al. [7] for a detailed review on scalar field models). Problem with scalar field models are two folds: the energy scale involved with such fields is $\sim 10^{-3}$ eV, which is not only much less than the energy scale of inflation but also much less than that of the supersymmetry (SUSY) breaking. Also, in order to achieve the slow-roll for this scalar fields (which in turn ensure the negative e.o.s), the mass of these fields have to be of the order of 10^{-33} eV, and this should not be corrected to higher values due to SUSY breaking. This is extremely difficult to achieve given our understanding of the hierarchy problem in the standard model of particle physics.

The above issues are addressed in the context of string theory and a robust model for quintessence was proposed by Panda et al. [8] (from now on PST model) which is based on the idea of axion monodromy in the context of Type-II B string theory. The quintessence field here is the axion field coming from the zero mode of two form and four form fields of Ramond-Ramond sector. When the shift symmetry associated with the axion fields is approximately broken, in a non-trivial way, the field develops a linear potential. This is perhaps the first controlled model for quintessence which is carefully constructed in the context of string theory. Subsequently the cosmological constraint on this model was derived by Gupta et al. [9], where it was showed that the observational data (coming mainly from the background Universe) allow the model to behave very close to the ΛCDM model when the initial value of the axion field is appropriately fixed. Changing this initial value can make deviation from the ΛCDM model.

Another challenge for any quintessence model is to keep the coupling of these scalar fields to the standard model fields sufficiently weak. This is to prevent the long range effect that these scalar fields can have (due to their small mass) in our Solar system. In the PST model, this has been successfully achieved since the axion field from the Ramond-Ramond sector does not interact with the fields of the standard model sector. But such an axion field, in principle, can interact with dark matter sector of the Universe. At present, our understanding of the dark matter sector is very limited, specifically its interaction with other fields. Currently, the dark matter is assumed to be weakly interacting matter particles (WIMP). Thus it is plausible to assume that the axion field, in the present context, can interact with the dark matter sector. This interaction could be also of non-perturbative nature.

In this paper, we investigate the cosmology with the PST-model of quintessence where the axion field is cou-

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pled with the dark matter sector of the theory but does not couple with visible matter sector of the Universe. We first study the background evolution in this set up and discuss the possible modifications that these coupling can result in the evolution of the Universe. We also study the evolution of the matter perturbation in linear theory. Finally we use different observational data to study the constraint on different parameters in our model.

In the next section, we formulate the equations for the background evolution of the Universe assuming the axion field is coupled to the dark matter sector of the Universe. In section III, we formulate the matter perturbation the linear regime in our model. We discuss the evolution of the linear growth of the matter fluctuations. In section IV, we discuss the observational constraints in our model using various observational data. Finally we conclude in section V.

II. BACKGROUND EVOLUTION WITH COUPLED AXION

We start with an interacting picture where the axion field is coupled with the dark matter sector (d) of the Universe. The visible matter sector (b) is not coupled with the axion field. The relevant equations are given by

$$\begin{aligned} \ddot{\phi} + \frac{dV}{d\phi} + 3H\dot{\phi} &= C(\phi)\rho_d \\ \dot{\rho}_d + 3H(\rho_d) &= -C(\phi)\rho_d\dot{\phi} \\ \dot{\rho}_b + 3H(\rho_b) &= 0 \end{aligned} \tag{1}$$

$$H^2 = \frac{\kappa^2}{3}(\rho_b + \rho_d + \rho_\phi) \tag{2}$$

together with the flatness condition

$$1 = \frac{\kappa^2\rho_b}{3H^2} + \frac{\kappa^2\rho_d}{3H^2} + \frac{\kappa^2\dot{\phi}^2}{6H^2} + \frac{\kappa^2 V(\phi)}{3H^2} \tag{3}$$

Here $C(\phi)$ represents coupling parameter between the axion field and dark matter. Due to our ignorance about the detail physics for this underlying interaction between the axion field and the dark matter, we address this issue at a phenomenological level and assume $C(\phi)$ to be a constant following the earlier work by Amendola [10]. For $C = 0$ we recover the uncoupled case.

Next, we define the following dimensionless variables:

$$x = \frac{\kappa\dot{\phi}}{\sqrt{6}H},\ y = \frac{\kappa\sqrt{V(\phi)}}{\sqrt{3}H}$$
$$z = \frac{\kappa\sqrt{\rho_b}}{\sqrt{3}H},\ \lambda = \frac{-1}{\kappa V}\frac{dV}{d\phi}\ \Gamma = \frac{V\frac{d^2V}{d\phi^2}}{\left(\frac{dV}{d\phi}\right)^2} \tag{4}$$

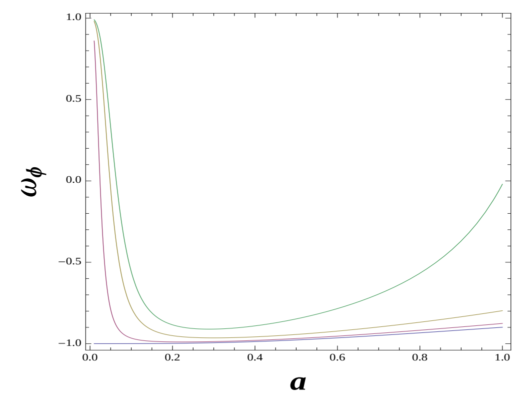

FIG. 1: Behaviour of the equation of state for the axion field. Bottom to Top: $W = 0, 0.01, 0.03, 0.06$. $\Omega_{d0} = 0.23, \Omega_{b0} = 0.05, \lambda_i = 0.7$.

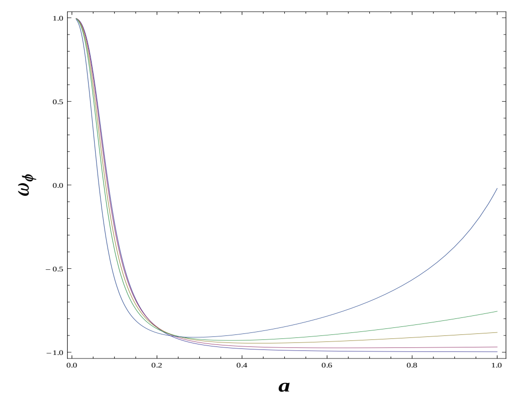

FIG. 2: Behaviour of the equation of state for the axion field. Bottom to Top: $\lambda_i = 0.05, 0.3, 0.5, 0.6, 0.7$. $\Omega_{d0} = 0.23, \Omega_{b0} = 0.05, W = 0.06$.

The density parameter Ω_ϕ and the equation of state for the axion field w_ϕ are given as,

$$\Omega_\phi = x^2 + y^2 \tag{5}$$

$$\gamma = 1 + w_\phi = \frac{2x^2}{x^2 + y^2} \tag{6}$$

With this, one can form an autonomous system of equations:

$$\begin{aligned} \Omega'_\phi &= W\sqrt{3\gamma\Omega_\phi}(1 - \Omega_\phi - z^2) + 3\Omega_\phi(1 - \Omega_\phi)(1 - \gamma) \\ \gamma' &= W\sqrt{\frac{3\gamma}{\Omega_\phi}}(1 - \Omega_\phi - z^2)(2 - \gamma) + \lambda\sqrt{3\gamma\Omega_\phi}(2 - \gamma) \\ &\quad -3\gamma(2 - \gamma) \\ z' &= -\frac{3}{2}z\Omega_\phi(1 - \gamma) \\ \lambda' &= \sqrt{3\gamma\Omega_\phi}\lambda^2(1 - \Gamma), \end{aligned} \tag{7}$$

where $W = \frac{C}{\kappa}$. We evolve the above system of equations from the decoupling era ($a = 10^{-3}$) to the present day

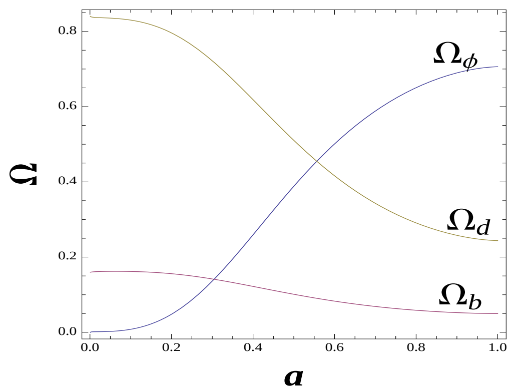

FIG. 3: Behaviour of the density parameters for different components. $\Omega_{d0} = 0.23, \Omega_{b0} = 0.05, \lambda_i = 0.7, W = 0.06$.

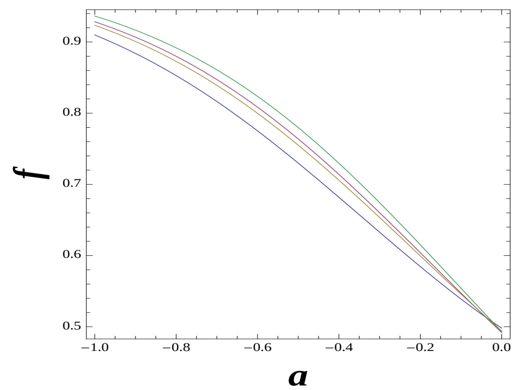

FIG. 4: Behaviour of the growth factor $f = \frac{d(log\delta_t)}{d(loga)}$ as a function of the scale factor. From top to bottom, ΛCDM, $W = 0, 0.03, 0.06$. $\Omega_{d0} = 0.23$ and $\Omega_{b0} = 0.05$. $\lambda_i = 0.7$ for models different from ΛCDM.

($a = 1$). Given the initial conditions for γ, Ω_ϕ z and λ, we can solve the system. The scalar field is initially frozen due to large Hubble damping, and this fixes the initial condition for γ, $\gamma_i \approx 0$. The initial value for λ, λ_i, is a parameter in our model. λ_i determines the deviation of w_ϕ from the initial $w_\phi \sim -1$ frozen state as the universe evolves with time. For smaller λ_i, the deviation extremely small, and the scalar field behaves as a cosmological constant at all time. For larger values of λ_i, w_ϕ has large deviation from -1 as the universe evolves. In general, the contribution of scalar field to the total energy density is negligible in the early universe. Nevertheless one has to fine tune the initial value of Ω_ϕ in order to have its correct contribution at present. In this regard, $\Omega_\phi(initial)$ is related to the $\Omega_\phi(z = 0)$. Similarly the initial value of z (which is related to the density parameter for visible matter) can be related to the present day value of the Ω_b which is fixed at $\Omega_{b0} = 0.05$ in our subsequent calculations.

In our study the scalar field is the axion field constructed using axion monodromy in the Type-IIB string theory. the form of the potential for this field is given by

$$V(\phi) = \frac{\mu^4}{f_a}\phi, \tag{8}$$

where

$$f_a^2 = \frac{g_s^2 M_{pl}^2}{6L^4},\ \mu^4 = \mu_1 + \mu_2. \tag{9}$$

Here $\mu_1 = \frac{2e^{4A_0}}{(2\pi)^5 g_s \alpha'^2}$ and $\mu_2 = cM_{SB}^4 e^{2A_0}\left(\frac{R^2}{\alpha' L^4}\right)$. g_s is the string coupling constant and $V_1 = L^6\alpha^{'3}$ is the volume of the internal space in the ten dimensional space time. A_0 is related to the warp factor at the location of NS5 brane and R is the radius of the Ads like throat (See [8] for the detail construction of this model). It can be shown that the μ_2 term have the dominant contribution in the potential $V(\phi)$. With this form of the potential, $\Gamma = 0$ in the system of equations (7). In figure 1 and 2, we show the behaviour of the equation of state for the axion field w_ϕ as a function of redshift keeping λ_i fixed and varying W (figure 1) and keeping W fixed at varying λ_i (figure 2). From these figures it is clear that w_ϕ deviates more from $w = -1$ for larger λ_i as well as for larger W. For smaller values of these two parameters, w_ϕ behaves very close to the Λ value $w = -1$. In figure 3, we show the behaviour of the density parameters for different components as a function of scale factor. It is evident that in the early era, Universe is mostly dominated by the visible as well as dark matter components. But as the Universe evolves, the axion field starts dominating in the late epoch.

We mention that the axion field is initially frozen due to large Hubble damping and that sets the initial $w_{\phi i} = -1$. But there is sudden jump initially in the w_ϕ from this frozen value as can be seen from the figures 1 and 2. This is due to the coupling between the axion field and dark matter component. But it does not create any pathology in Universe's evolution; because during the time when this sudden jump happens the contribution from the axion field to the total energy budget of the Universe is negligible. So the axion field does not affect the Universe's evolution during this time. We should point that similar things happens when a thawing scalar field is coupled to the total matter component (visible + dark matter) of the Universe [12].

III. LINEAR PERTURBATION

In this section, we write down the equations that governs the growth of fluctuations for dark matter and visible matter. In our model the dark matter is coupled to the axion field whereas the the visible matter is uncoupled.

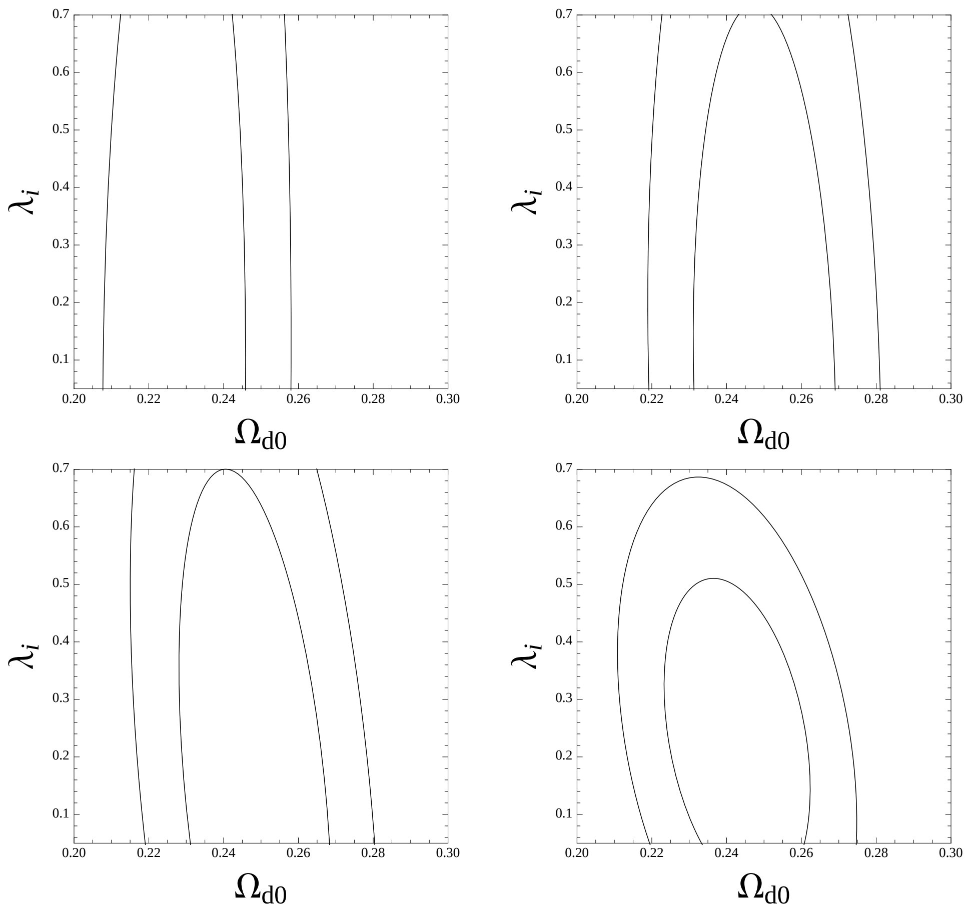

FIG. 5: 68% and 95% confidence contours in $\Omega_{d0} - \lambda_i$ plane. Top Left: uncoupled case $W = 0$, Top Right: coupled case $W = 0.01$, Bottom Left: coupled case $W = 0.03$, Bottom Right: coupled case $W = 0.06$.

We work in the longitudinal gauge,

$$ds^2 = a^2 \left[-(1+2\Phi)d\tau^2 + (1-2\Psi)dx^i dx_i \right], \tag{10}$$

where τ is the conformal time and Φ and Ψ are the two gravitational potential. It is well known that in the absence of any anisotropic stress $\Phi = \Psi$. We follow the prescription given by Amendola [10] and write the equations for the density perturbations for visible matter and dark matter in the Newtonian limit which is valid for small scales. In this scale, one can safely assume that the axion field does not cluster otherwise it will behave like a massive dark matter. With these assumptions, the equations governing the growth of the linearised fluctuations in dark matter and visible matter are given by:

$$\delta_d'' + \left(1 + \frac{\mathcal{H}'}{\mathcal{H}} - 2\beta_d x\right)\delta_d' - \frac{3}{2}(\gamma_{dd}\delta_d\Omega_d + \gamma_{db}\delta_b\Omega_b) = 0 \tag{11}$$

$$\delta_b'' + \left(1 + \frac{\mathcal{H}'}{\mathcal{H}} - 2\beta_b x\right)\delta_b' - \frac{3}{2}(\gamma_{db}\delta_d\Omega_d + \gamma_{bb}\delta_b\Omega_b) = 0 \tag{12}$$

Here $\gamma_{ij} = 1 + 2\beta_i\beta_j$ and $\mathcal{H}$ is conformal Hubble parameter $\mathcal{H} = aH$. For our case, $\beta_b = 0$ and $\beta_d = W$. The density fluctuations δ_d and δ_b are defined as $\delta_i = \frac{\delta\rho_i}{\rho_i}$. The total density fluctuation for the matter component (dark + visible) is given by

$$\delta_t = \frac{\delta_d\Omega_d + \delta_b\Omega_b}{\Omega_d + \Omega_b}. \tag{13}$$

In figure 4, we show the behaviour of the growth function $f = \frac{d(log\delta_t)}{d(loga)}$ as a function of scale factor. One can see that the growth is suppressed as one increases the coupling constant W.

IV. OBSERVATIONAL CONSTRAINTS

In this section, we use the current observational data to put constraints on our model (see [11] for earlier works on observational constraints for coupled quintessence).

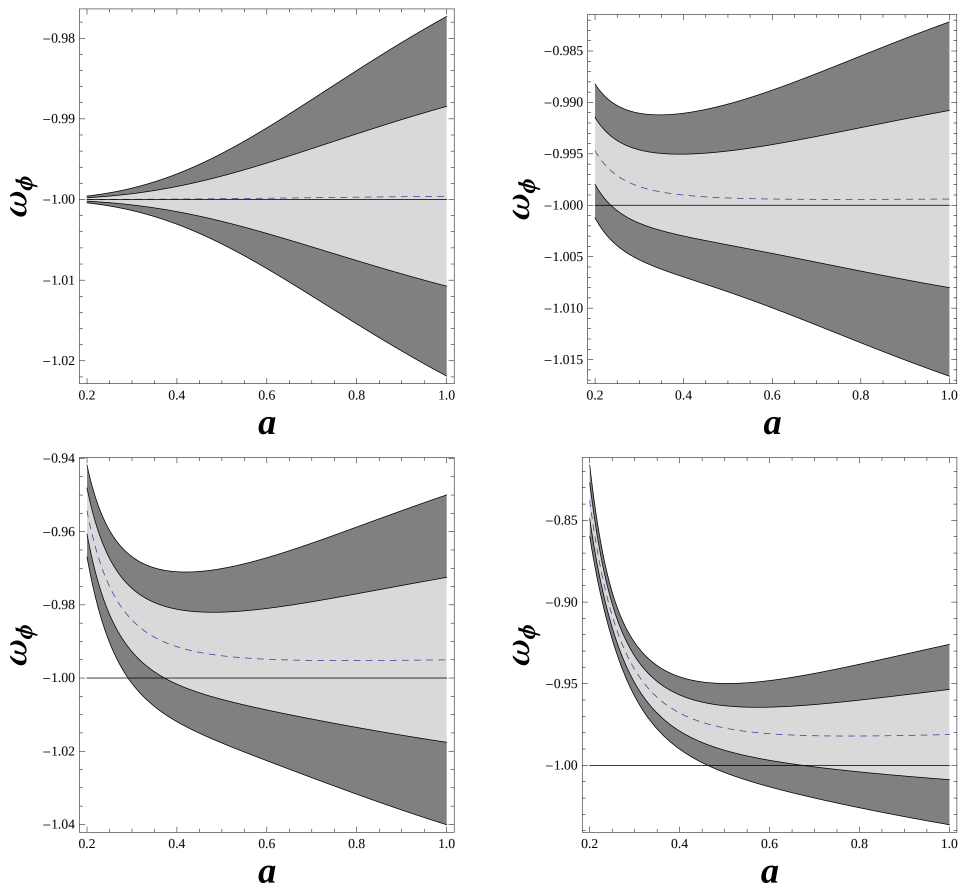

FIG. 6: Allowed behaviour for the equation of state of the axion field w_ϕ at 68% and 95% confidence level. Top Left: uncoupled case $W = 0$, Top Right: coupled case $W = 0.01$, Bottom Left: coupled case $W = 0.03$, Bottom Right: coupled case $W = 0.06$. Light Shade for 68% and dark shade for 95%. The dashed line represents the best fitted behaviour and the solid line represents line represents the phantom divide.

We start with the Supernova Type-Ia observations which directly probes the cosmological expansion. It actually measures the luminosity distance of different supernova explosions at different redshifts which is defined as:

$$d_L(z) = (1+z)\int_0^z \frac{1}{H(z')}dz'. \tag{14}$$

The actual observable, *distance modulus* is given by

$$\mu = 5log\frac{d_L}{Mpc} + 25. \tag{15}$$

We consider the Union2.1 compilation containing 580 data points for μ at different redshifts [13].

Next, we use the observational constraints on Hubble parameter which have been recently compiled by Moresco et al. [14] in the redshift range $0 < z < 1.75$ using the differential evolution of the cosmic chronometers. They have built a sample of 18 observational data point for $H(z)$ spanning almost 10 Gyr of cosmic evolution.

Next, we use the combined BAO/CMB constraints as derived recently by Giostri et al. [18]. This gives the constraint on the angular scales for the Baryon Acoustic Oscillation peak in the matter power spectrum. At present we have measurements from SDSS survey [16] , 6dF Galaxy Survey [15] and more recently, by the WiggleZ team [17]. The corresponding covariance matrix is given by Giostri et al [18].

We further use the measurement for the growth parameter $f = \frac{dlog\delta_t}{d\log a}$. The list of all the currently available growth data is given in the reference [19] (see also references therein).

With this, we calculate the combined likelihood:

$$-2\log\mathcal{L} = -2\log(\mathcal{L}_{sn}\times\mathcal{L}_{bao}\times\mathcal{L}_{hub}\times\mathcal{L}_{gr}). \tag{16}$$

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This likelihood function is a function of the model parameters Ω_{d0}, λ_i and W. As we mentioned earlier, we have fixed $\Omega_b = 0.05$ for the visible matter. We maximize the likelihood function for $W = 0$ for the uncoupled case and $W = 0.01, 0.03, 0.06$ for the coupled case and draw the corresponding confidence contours in the $\Omega_{d0} - \lambda_i$ plane where $\Omega_{m0} = \Omega_{d0} + \Omega_{b0}$.

In figure 5, we show the confidence contours in the $\Omega_{d0} - \lambda_i$ plane for different values of the coupling constant W. We fix $\Omega_{b0} = 0.05$ for our calculations. The first thing to be noticed is that, for the uncoupled case ($W = 0$), Ω_{d0} and λ_i are completely uncorrelated. As one increases the value of W, the two parameters become slightly anti correlated. One concludes from these plots that in the coupled case the upper bound on the initial slope of the axion potential λ_i is comparatively smaller than the uncoupled case. As the value of the axion field is inversely proportional to the slope of the potential in our model, this translates to a bigger lower bound for the axion field in the coupled case.

We have discussed in the earlier section that the deviation of the cosmological evolution from the concordance ΛCDM behaviour is controlled by the two parameters λ_i and W. Hence we use the standard error propagation technique [20] to calculate the error in the equation of state w_ϕ knowing the errors in λ_i and Ω_{d0} for different values of W. The results are shown in the figure 6. In these figures, we also show the region in the phantom side as obtained by the error propagation. But for our axion field the relevant region is above the line $w_\phi = -1$. We observe from these plots that for the uncoupled case, the allowed behaviour for w_ϕ is very close to the Λ line $w = -1$. This is consistent with the earlier results obtained by Gupta et al. [9]. As one increases the coupling parameter W, larger deviation from $w = -1$ is allowed. This confirms that the coupling with dark matter can result the axion-quintessence field to behave differently from the cosmological constant Λ.

V. CONCLUSION

The PST model proposed Panda et al. is perhaps the first controlled model for quintessence, built in the context of string theory using the idea of axion monodromy. In this set up, the axion field arises in the Ramond-Ramond sector of string theory avoiding the interaction with standard model fields. But such an axion field can, in principal, interact with the dark matter sector of the Universe. In this work, we study the cosmology in a PST model where the axion field is coupled with dark matter sector but not with the visible matter sector. Due to the lack of understanding about the detail physics for this interaction, we assume the coupling function to be a constant for simplicity. This kind of coupling function has been earlier used by Amendola and others for studying the coupled quintessence models.

In our model, the axion field is initially frozen due to large Hubble damping and the equation of state $w_\phi \sim -1$ initially. But as the Universe evolves, w_ϕ deviates from this frozen value. This deviation is larger for larger coupling and larger initial slope of the potentials λ_i. Next we set up the equation of for linear matter density perturbation in our model and study the evolution of the growth function showing that stronger coupling suppresses the growth.

Finally we use the latest data from Supernova Type-Ia, Baryon acoustic oscillations, measurements of the Hubble parameter as well as the measurement of the growth, to constrain the model parameters. For the uncoupled case, we confirm the earlier results by Gupta et al.[9] that data allow extremely small deviation from the cosmological constant Λ. But as one introduces the coupling between the axion field and the dark matter, larger deviation from the $w = -1$ behaviour is allowed with increasing strength of the coupling.

VI. ACKNOWLEDGEMENT

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[1] A. G. Riess et al., Astron. J. **116**, 1009 (1998); S. Perlmutter et al., Astrophys. J. **517**, 565 (1999); J. L. Tonry et al., Astrophys. J. **594**, 1 (2003).

[2] A. Melchiorri et al., Astrophys. J. Lett. **536**, L63 (2000); A. E. Lange et al., Phys. Rev. D **63**, 042001 (2001); A. H. Jaffe et al., Phys. Rev. Lett. **86**, 3475 (2001); C. B. Netterfield et al., Astrophys. J. **571**, 604 (2002); N. W. Halverson et al., Astrophys. J. **568**, 38 (2002).

[3] S. Bridle, O. Lahab, J. P. Ostriker and P. J. Steinhardt, Science **299**, 1532 (2003); C. Bennett et al., Astrophys. J. Suppl. Ser. **148**, 1 (2003); G. Hinshaw et al., Astrophys. J. Suppl. Ser. **148**, 135 (2003); A. Kogut et al., Astrophys. J. Suppl. Ser. **148**, 161 (2003); D. N. Spergel et al., Astrophys. J. Suppl. Ser. **148**, 175 (2003).

[4] D. Eisenstein, Astophys. J., **633**, 560 (2005).

[5] Miao Li, Xiao-Dong Li, Shuang Wang, arXiv:1103.5870; V. Sahni and A. A. Starobinsky, Int. J. Mod. Phys. D **9**, 373 (2000); S. M. Carroll, Living Rev. Rel. **4**, 1 (2001); P. J. E. Peebles and B. Ratra, Rev. Mod. Phys. **75**, 559 (2003); T. Padmanabhan, Phys. Rept. **380**, 235 (2003).

[6] B. Ratra and P.J.E. Peebles, Phys. Rev. D, **37**, 3406 (1988); M.S. Turner and M. White, Phys. Rev. D, **56**, R4439 (1997) R.R. Caldwell, R. Dave, and P.J. Steinhardt, Phys. Rev. Lett. **80**, 1582 (1998); A.R. Liddle and R.J. Scherrer, Phys. Rev. D, **59**, 023509 (1999); P.J. Steinhardt, L. Wang, and I. Zlatev, Phys. Rev. D, **59**, 123504 (1999); R. J. Scherrer and A. A. Sen, Phys. Rev. D, **77**, 083515 (2008)

[7] E. J. Copeland, M. Sami and S. Tsujikawa, Int. J. Mod. Phys. D **15**, 1753 (2006)

[8] S.Panda, Y.Sumitomo, & S. P.Trivedi, Phys. Rev. D, **83**, 083506, (2011).
[9] G. Gupta, S. Panda and A. A. Sen, Phys. Rev. D, **85**, 023501 (2012).
[10] L. Amendola, Phys. Rev. D, **69**,103524 (2004); L. Amendola, Phys. Rev. D, **62**, 043511 (2000); C. D. Porto, L. Amendola, Phys. Rev. D, **77**, 083508 (2008)
[11] S. Lee, Guo-Chin Liu, Kin-Wang Ng, Phys. Rev.D, **73**, 083516 (2006); S. Lee, Guo-Chin Liu, Kin-Wang Ng, Phys. Rev. D, **81**, 061302 (2010).
[12] G.Gupta, S.Majumdar and A.A.Sen, Mon. Not. Roy. Astron. Soc., **420**, 1309 (2012).
[13] N. Suzuki et al., Astrophysics J. **746**, 85 (2012).
[14] M. Moresco,L. Verde, L. Pozzetti, R. Jimenez, A. Cimatti, arXiv:**1201.6658**.
[15] F. Beutler et al., Mon. Not. Roy. Astron. Soc., **416**, 3017 (2011).
[16] W. J. Percival et al., Mon. Not. Roy. Astron. Soc., **401**, 2148 (2010).
[17] C. Blake et al., Mon. Not. Roy. Astron. Soc., **418**, 1707 (2011).
[18] R. Giostri, M. V dos Santos, I. Waga, R. R. R. Reis, M. O. Calvao, B. L. Lago JCAP, **1203**, 027 (2012).
[19] G. Gupta, S. Sen and A. A. Sen, JCAP, **1204**, 028 (2012).
[20] D. Coe , arXiv:0906.4123 [astro-ph.IM]